LINEAR EVOLUTION OF PERTURBATIONS IN BOUNDARY LAYERS WITH A VELOCITY PROFILE INFLECTION

M. B. Zel'man and B. V. Smorodskii UDC 532.517

The vorticity distribution of the main flow is of importance for the nature of the laminar-turbulent transition (LTT). The presence of extrema (inflections in the mean velocity profile) induces specific singularities in both the laminar motion stability parameters and in the structure of the turbulent domain being investigated. This is shown especially clearly in the example of free shear flows [1-3].

The appearance of profile inflections in boundary layers can be caused by the geometry of the surface being streamlined, the action of large-scale incoming flow configurations, or the self-action of intense instability waves within the layer. In this latter case the occurrence of a singularity, as it is assumed, induces the development of a secondary high-frequency instability resulting in turbulization. The papers [3-9] are devoted to a study of these problems. The stability of model or secondary flows extracted from experiments was considered in [3, 7-9] within the framework of a linear inviscid approximation. In a more complete formulation (nonzero viscosity), computations are performed for examples of pre-separation boundary layers [4, 6]. A detailed investigation of the influence of a small deformation (in the form of a localized jet) on the flow stability in a plane channel is performed in [10].

However, the problem of seeking general dependences of the spectrum and structure formation of instable perturbations in a boundary layer on the location and degree of the inflection remains open. Its solution for a range of Reynolds numbers and fluctuation wave spectra realized in typical LTT experiments is attempted in this paper. Within the framework of a linear locally parallel stability problem, the evolution of waves and fluctuation wave packets of a discrete and a continuous spectrum is considered during variation of the inflection parameters. The universality of regularities disclosed in the basis flow [11] is established by comparison with typical profiles of other kinds.

We represent the boundary layer velocity field in the local parallelism approximation in the form $\mathbf{U} = (U + \varepsilon u_1, \varepsilon u_2, \varepsilon u_3)$, where $(U(y), 0, 0)$ corresponds to the main flow while $\varepsilon(u_1, u_2, u_3)$ is its perturbation. We select $U = U_G(y)$ as basis [11], represented by the curve 2 in Fig. 1. Such a profile simulates the motion of intense vortices in the layer and in the variables, which are dimensionless with respect to the thickness of the displacement and the incoming flow velocity, it is given by the dependence

$$U_{\mathrm{G}} = \begin{cases} U_- + \varkappa(\operatorname{th}\tilde{y} + 1), & y < y_r, \\ U_+ + \varkappa(\operatorname{th}\tilde{y} - 1), & y > y_r, \end{cases} \qquad \tilde{y} = \frac{y - y_r}{\delta}, \; \delta = \mathrm{Re}^{-1/2} \ll 1, \; y_r \gg \delta. \tag{1}$$

Here Re is the stream Reynolds number, y_r is the location of the inflection point, δ and $\varkappa$ characterize the zone width and the degree of profile deviation from the Blasius distribution U_B (curve 1, Fig. 1), $U_\pm$ are solutions of the boundary value problem

$$\frac{\partial^3\Psi}{\partial y^3} + \frac{\Psi}{2}\frac{\partial\Psi}{\partial y} = 0, \; \Psi = \{\Psi_-, \Psi_+, \Psi_{\mathrm{B}}\}, \; U_{\mathrm{B}} = \frac{\partial\Psi}{\partial y},$$
$$0 \leqslant y \leqslant y_r, \; U_- = \partial\Psi_-/\partial y, \; \Psi_-(0) = U_-(0) = 0,$$
$$U_-(y_r) = U_{\mathrm{B}}(y_r) - \varkappa, \; y_r \leqslant y < \infty, \; U_+ = \partial\Psi_+/\partial y,$$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 50-55, January-February, 1991. Original article submitted June 2, 1989; revision submitted September 1, 1989.

 0021-8944/91/3201-0046$12.50

$$\Psi_+(y_r) = \Psi_-(y_r),\ U_+(y_r) = U_B(y_r) + \varkappa,$$
$$U_+ \to 1 \text{ for } y \to \infty. \tag{2}$$

Besides being of independent interest, the selection of U_G as the basis profile is due to the simplicity of the dependence $U_G = U(y, y_r, \varkappa, \delta)$.

The system of equations for the perturbations in a linear approximation takes the form

$$L(\eta) + \frac{\partial U}{\partial y}\frac{\partial u_2}{\partial z} = 0,\ L(\Delta u_2) - \frac{\partial^2 U}{\partial y^2}\frac{\partial u_2}{\partial x} = 0,$$
$$(\eta, u_2) = 0 \quad (y = 0),\ (\eta, u_2) \to 0 \quad (y \to \infty),\ (\eta, u_2) = (\eta^0, u_2^0) \quad (t = 0), \tag{3}$$
$$L = \frac{\partial}{\partial t} + U\frac{\partial}{\partial x} - \frac{\Delta}{\mathrm{Re}},\ \left|\frac{\partial U}{U\,\partial x}\right| \ll \lambda^{-1}$$

(η is the normal vortex component to the wall, and λ is the characteristic scale of the longitudinal perturbation change).

The solution of (3) is representable by the integral

$$u_j = \int_{-\infty}^{\infty} d\alpha \int_{-\infty}^{\infty} d\beta A(\alpha, \beta)\psi_j(y, \alpha, \beta)\exp i(\alpha x + \beta z - \bar{\omega}t) +$$
$$+ \int_{-\infty}^{\infty} d\alpha \int_{-\infty}^{\infty} d\beta \int_{-\infty}^{\infty} dk\, B(\alpha, \beta, k)\varphi_j(y, k)\exp i(\alpha x + \beta z - \tilde{\omega}t), \tag{4}$$

where $j = 1, 2, 3$, A and $B(\alpha, \beta, k)$ are found from the initial conditions, ψ_j, $\bar{\omega} = \bar{\omega}(\alpha, \beta) = \omega + i\omega_i$ are the eigenfunctions and values of the discrete spectrum of the Orr–Sommerfeld problem, and $\varphi_j(y, k)$, $\tilde{\omega}(\alpha, \beta, k)$ correspond to a continuous spectrum.

We shall examine the stability of U_G as a function of $\varkappa$, y_r, Re. According to computations for the Tollmien–Schlichting (TS) wave perturbations, the appearance of an inflection ($\varkappa > 0$) noticeably deforms the shape of $\psi_j(y)$ and the spectrum of the fluctuations being magnified. The nature of the influence is substantially related to $\varkappa$ and the location of the point y_r. It turns out that in a broad range of values of α, β, Re, corresponding to a linear instability domain of the undeformed flow, and interval of values $y_r = y \in \ell$ exists within which magnification of $\varkappa$ contributes both to broadening of the unstable wave spectrum and to growth of the maximum of their increments. The location of the interval ℓ and its dimensions vary with Re within $0.8 \le y \le 2.5$, enclosing the neighborhood of the wave critical layer $y = y_c \approx 1$ but not reaching the outer layer boundary $y \approx 5$ (see Fig. 1). It is essential that for an inflection located outside the domain ℓ, magnification of $\varkappa$ in this range of α, Re results in suppression of the instability. However, stabilization for an inflection in the near-wall domain $y < \ell$ is not global in nature. As $\varkappa$ grows, a shift of the instability zone occurs into the domain of HF vibrations without broadening of the interval α. Perturbation build-up for $y > \ell$ is not observed up to $10^{-3} \le \alpha \le 10^2$. This qualitatively distinguishes the process in the boundary layer from free shear flows that are always being destabilized by growth of $\varkappa$ [1, 2].

Dependences of the wave increments $\omega_i = \omega_i(\varkappa)$ for $F = 57.5 \cdot 10^{-6}$, Re = 625, $\beta = 0$ are presented in Fig. 2 for $y_r = 0.5, 0.75, 1.25, 1.75, 3.0$ (curves 1-5, respectively). It is seen that the domain of the critical layer $y_r \approx y_c$ is the boundary for ℓ: the increments drop for small $\varkappa < \varkappa_0$ into the domain of negative values of ω_i and grow monotonically for $\varkappa > \varkappa_0$. The deflection in the zone ℓ broadens the band of the unstable vibrations spectrum by shifting the maximally unstable vibrations into the high-frequency domain and touching the lower boundary slightly. The solid lines in Fig. 3 correspond to $\varkappa = 0$, $y_r = 1$; $\varkappa = 2\%$, $y_r = 1$; $\varkappa = 5\%$, $y_r = 2$; $\varkappa = 2\%$ for Re = 625, $\beta = 0$. The quantities $\omega_m = \omega(\omega_{im})$ and $\omega_{im} = \max_\omega \omega_i(R)$ for $\varkappa > 5\%$ can exceed the corresponding values for $\varkappa = 0$ by more than an order. Acute focusing of ψ_j in the layer $y \approx y_r$ is realized. Transformation of $|\psi_j|$ from the shape of the classical TS distribution through a two-hump structure to a sharp peak as $\varkappa$ increases is shown in Fig. 4. The curves 1-4 are obtained for Re = 625, $\beta = 0$, $\alpha = 0.15$ for $\varkappa = 0, 1, 3, 15\%$. For $\varkappa$ = const the frequency drop results in a certain flattening out of the $|\psi_1|$ peak, however, the domain of its localization remains invariant with respect to a change in

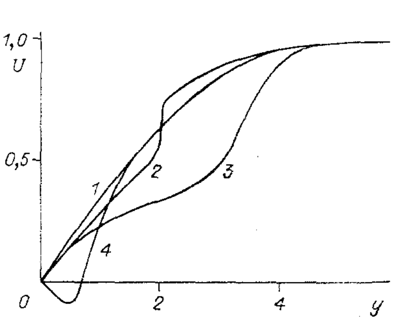

Fig. 1

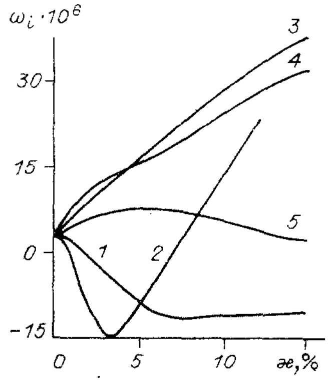

Fig. 2

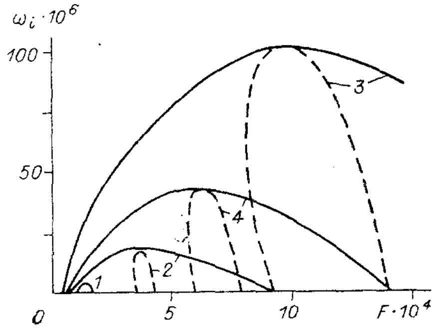

Fig. 3

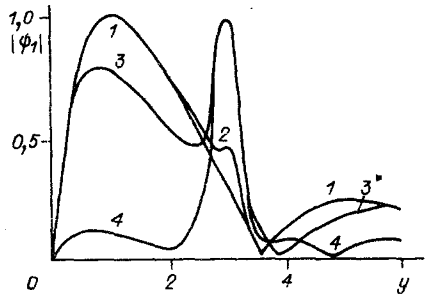

Fig. 4

ω. Consequently, the neighborhood of y_r becomes a zone of intensive fluctuation concentration of a broad spectrum band. At the same time, the change in $\varkappa$ is felt weakly by the dispersion TS dependence $\omega = \omega(\alpha, \beta)$. Despite the shift of $|\psi_1|$ from the zone of the critical layer y_c, the wave phase velocity $c = \omega/\alpha$ remains close to the value $U_B(y_c)$.

Therefore, local inflection changes $\varkappa > 0$ act substantially on the wave magnification velocity while its phase properties are determined by the boundary layer integral parameters. Let us note the influence of the degree of inflection $\varkappa$ on the instability zone width in Re ($\omega_i(\mathrm{Re}) > 0$). Magnification of $\varkappa$ (y_r = const) results in its extension. For $\varkappa > 0$ and Re $\to \infty$ $\omega_i \to \omega_{in}(y_r, \varkappa, \omega)$ (ω_{in} is the "inviscid" limit value). For the neutral stability curve Re(α) this denotes the emergence of the upper branch on the asymptote $\alpha \neq 0$. The computations results are represented in greater detail in [12]. In conformity with [10], perturbations with $\alpha > 5$ appear a property of the locality associated with the fact that outside a narrow neighborhood of y_c their amplitude drops abruptly to zero and remain invariant with respect to $\varkappa$ for an inflection at $y_r \neq y_c$. In the range $300 \leq \mathrm{Re} \leq 1500$, $0.4 \leq y \leq 4$, $\varkappa \leq 10\%$ the vibrations instability is due entirely to the TS-mode. Appearance of additional unstable modes were first detected for Re ≥ 2500.

Analysis of the stability of U_G with respect to three-dimensional ($\beta \neq 0$) perturbations qualitatively reproduces the dependences noted above: for $\varkappa > 0$ there is a domain $y_r \in \ell$ within which the eigenfunctions ψ_j are localized at y_r, and the unstable vibrations spectrum is broadened. An increase in the angles $\xi = \tan^{-1} \beta/\alpha$ of wave front propagation for ω = const lowers the magnification increment. As $\varkappa$ grows, the spread in the values of ξ corresponding to the unstable waves is expanded somewhat.

Focusing of the wideband wave motion in a narrow vertical layer $y \approx y_r$ noted above makes the question of the collective effects of perturbation evolution important. Analysis of the evolution of spatially localized perturbations can be reduced to examination of an approximate set of quasiharmonic packets. The solution of the problem here is to construct an asymptotic of the integral (4) for which a study of the relation $\bar{\omega} = \bar{\omega}(\alpha, \beta)$ is required over a field of complex α, β. Displacement of the path of integration of (4) from the real axes in the α, β planes with the requirement of convergence and passage through the saddle point α_0, β_0 ($\partial\bar{\omega}(\alpha_0, \beta_0)/\partial\alpha = x/t$, $\partial\bar{\omega}(\alpha_0, \beta_0)/\partial\beta = z/t$) taken into account results in representation of the solution in the form

$$u_j(y, \bar{\omega}, \mathrm{Re}) \sim (1/t)\psi_j(y, \alpha_0, \beta_0, \mathrm{Re}) \exp i(\alpha_0 x + \beta_0 z - \bar{\omega}_0 t). \quad (5)$$

Application of the mentioned approach to U_G disclosed that qualitative singularities of the group wave evolution remain analogous to those found for U_B [13]. In particular, contraction of the range of packet carrier frequencies that grow in the system $x_1 = x - t\,\partial\bar{\omega}/\partial\alpha$ ($\gamma = \mathrm{Im}(\bar{\omega} - \alpha\partial\bar{\omega}/\partial\alpha)$) with respect to the corresponding domains of unstable monochromatic vibrations occurs [the dashed lines 2-4 in Fig. 3; the computation parameters correspond to those used for monochromatic waves (solid lines 2-4)]. By virtue of the narrowness of the ω-instability bands ($\gamma > 0$) it can be expected that during evolution the wideband background perturbation is transformed into a wave tooth variable carrier frequency $\omega_m = \omega(\omega_{im}(\mathrm{Re}))$, $\omega_{im} = \max_\omega \omega_i(\mathrm{Re})$ [13]. The singularity of the inflectional $U_G(y)$ for $\varkappa > 0$ is an increase in the values of ω_m, γ_m and transformation of the tooth into a quasiharmonic packet as $\mathrm{Re} \to \infty$, when $\gamma_m \to \gamma_{in}$ and $\omega_m \to$ const as was noted above.

Commonality of the singularities detected for U_G needs clarification. To this end, a flow generalizing to the case of arbitrary $\delta \gtreqless \mathrm{Re}^{-1/2}$, $y_r \gtreqless \delta$ and approximating the preseparation U_S and K-transition (secondary) U_K profiles (see lines 4 and 3 in Fig. 1) was examined. The last two distributions were constructed by a smooth deviation to the second derivative by polynomials of the initial profile $U_B(y)$. The maximal magnitude of the deviation of the profiles under consideration from the distribution $U_B(y)$ was here selected as the parameter $\varkappa$ (degree of inflection). It turns out that within the framework of the generalized U_G of the flow the shift $y_r \to 0$ (in the viscous sublayer domain) does not cause localization of the peak $|\psi_1|$ in $y \approx y_r$. The flow is stabilized since y_r becomes below the layer ℓ. Extension of the zone δ for deflections $y_r \in \ell$ appears weakly in the spectrum characteristics but results in smoothing of the peak $|\psi_1|$ in the neighborhood of y_r.

The perturbation evolution properties detected in the U_G flow are reproduced qualitatively in the U_K flow. A vertical layer ℓ ($0.8 \lesssim y \lesssim 3$) exists within which the inflection ($\varkappa > 0$) results in broadening of the unstable fluctuation spectrum, and in its shifting to the domain of large ω, α, ω_i and focusing of $|\psi_1|$ at y_r.

The appearance of reverse flow domains in the stream (U_S type profile) changes a number of the singularities remarked above. The presence of an inflection for $y_r \to 0$ does not hinder formation of the $|\psi_1|$ peak near the wall more while the spectrum of unstable fluctuations is shifted weakly into the domain of large ω, α. The computations performed agree with the data in [4, 6]. Therefore, universality of the transition effects detected during analysis of U_G turns out to be limited by the requirement of monotonicity of the U(y) profile. Such conditions are typical in boundary layers on a flat plate where, therefore, the formation of an inflection in the interval $y_r \in \ell$ should contribute to accelerated emergence in the turbulent mode because of broadening of the frequency-wave spectrum and the fluctuation velocities.

In this context, confirmation is found for the idea of the secondary instability as a transition mechanism [7, 9]. In particular, generation of localized turbulence domain (spots) can be explained by the modulation of U_K that causes wandering of the inflection parameters ($\varkappa$, y_r) = f(x, y, z, t) (such modulation actually holds [9]). Then the vertical focusing of the broadband fluctuations supplemented their (x, z, t) localization taken with the modulation velocity U_K.

The "projections" observable on oscillograms are associated with the development of this process during a K-transition [9]. However, our computations do not confirm such a relation since the domain for recording "projections" turns out to be above the boundary $y_r \in \ell$ beyond which amplification does not occur. The deduction of the "non-secondary" nature of the flare-ups [14] finds a foundation. Nevertheless, the possibility of initiation of a transition because of the secondary instability mechanism (for $y_r \in \ell$) is quite probable.

In conclusion, we examine the influence of the inflection on the continuous spectrum mode. The question of susceptibility of the boundary layer to perturbations in the incoming stream, particularly to continuous spectrum waves, remains open and it can be assumed that one of the mechanisms of action on the transition in the boundary layer is interaction of the mentioned and the TS modes. In the case of a Blasius flow such interaction turns out to be inefficient because of TS localization with the boundary layer, where the continuous spectrum fluctuations have practically zero amplitude. The interaction coefficients dependent on the product of amplitude functions turn out to be close to zero.

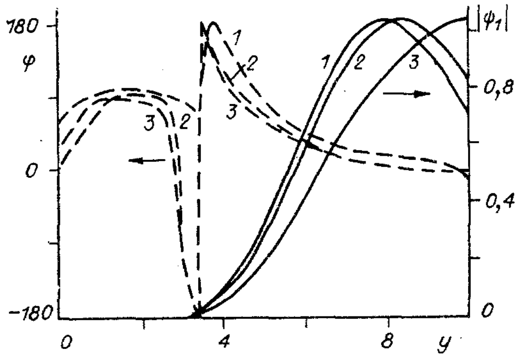

Fig. 5

The appearance of an inflection in the mean velocity profile causes localization of the TS wave fluctuations at the height $y \approx y_r$, and in the case of an analogous focusing action on the continuous spectrum mode the appearance of a finite contribution to these coefficients could be expected. The dependence of continuous spectrum fluctuations of the linear problem [15] on $\varkappa$, y_r, Re, α, ω for the profiles U_G, U_S, U_K was studied in this paper to analyze the efficiency of the process. It turns out that localization of continuous spectrum fluctuations do not occur at the inflection point in the studied range of parameters ($300 \le \mathrm{Re} \le 1000$, $\varkappa_G \le 10\%$, $\varkappa_K \le 30\%$, $y_r \le 4.5$, $20\cdot10^{-6} \le F \le 115\cdot10^{-6}$). Dependences of the vorticity wave amplitude and phases (continuous and dashed lines, respectively) on y are represented in Fig. 5 for the velocity profiles 1-3 for Re = 625, $\alpha = 0.17$, U_B; U_G, $\varkappa = 6\%$, $y_r = 3.5$; U_K, $\varkappa = 30\%$, $y_r \approx 3.5$. Conversely, the first maximum of the wave as $\varkappa$ grows is easily displaced into the outer domain of the layer. Only reconstruction of the transverse phase distribution $\varphi = \arg\psi_1(y)$ is noticeable in the inner domain. Such behavior (absence of amplitude localization of different kinds of modes in the velocity profile inflection zone) apparently makes the proposed mechanism of external perturbation action inefficient. The appearance of inflections in the mean velocity profile is a source of transformation of the fluctuation evolution within the layer.

LITERATURE CITED

1. M. M. Sushchik, "Dynamics of structures in shear flow," in: Nonlinear Waves: Structures and Bifurcations [in Russian], Nauka, Moscow (1987).
2. C. Ho and P. Huerre, "Perturbed free shear layers," Ann. Rev. Fluid Mech., 16, 305 (1984).
3. R. Betchov and W. Criminale, Jr., Questions of Hydronamic Stability [Russian translation], Mir, Moscow (1971).
4. A. H. Nayfeh, S. A. Ragab, and A. Al-Maaitah, Effect of Roughness on the Stability of a Boundary Layer, AIAA Paper No. 86-1044, New York (1986).
5. J. M. Kendall, Experimental Study of Laminar Boundary Layer Receptivity to a Traveling Pressure Field, AIAA Paper No. 87-1257, New York (1987).
6. K. Gruber, H. Bestek, and H. Fasel, Interaction between a Tollmien–Schlichting Wave and a Laminar Separation Bubble, AIAA Paper No. 87-1256, New York (1987).
7. H. F. Greenspan and D. J. Benny, "Shear-layer instability, breakdown, and transition," J. Fluid Mech., 15, Part 1 (1963).
8. S. Ya. Gertsenshtein, "Stability of a nonstationary rectilinear plane-parallel ideal fluid flow," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 2 (1969).
9. M. Nishioka, M. Asai, and S. Iida, "An experimental investigation of the secondary instability," in: Laminar-Turbulent Transition IUTAM Symp. Stuttgart, Germany (1979), Springer, Berlin (1980).
10. M. A. Gol'dshtik and V. N. Shtern, Hydrodynamic Stability and Turbulence [in Russian], Nauka, Novosibirsk (1977).
11. T. B. Gatsky, "Vortex motion in real bounded viscous flow," Proc. R. Soc. London, A397, 397 (1985).
12. M. B. Zel'man and B. V. Smorodskii, "Development and interaction of perturbations in boundary layers with velocity profile inflection," Preprint No. 2-89 [in Russian], Inst. Teor. Prikl. Mekh., Akad. Nauk SSSR, Sib. Otd., Novosibirsk (1989).
13. M. B. Zel'man and B. V. Smorodskii, "Wave perturbation packets of Blasius flow," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1 (1988).

14. Yu. S. Kachanov, "Resonance wave nature of the transition to turbulence in a boundary layer," in: Modeling in Mechanics [in Russian], Vol. 1, No. 2 (1988).
15. V. N. Zhigulev and A. M. Tumin, Origin of Turbulence [in Russian], Nauka, Moscow (1987).

COMPUTING THE COMPRESSIBLE LAMINAR BOUNDARY LAYER ON A SHARP BODY OF BIELLIPTIC SECTION

V. N. Vetlutskii

UDC 532.526

The problem of determining the parameters of a three-dimensional boundary layer is very pertinent, since its solution gives the distribution of friction and heat flux at the surface of the immersed body. From numerical solution of the full boundary layer equations we find the flow parameters: velocity components, temperature and density, from which one obtains new knowledge of the whole flow picture.

Most papers on computing the laminar three-dimensional boundary layer deal with incompressible flow [1-5]. At supersonic speed of the incident flow it has been studied most frequently on blunt bodies (see, e.g., [6-9]). The three-dimensional compressible boundary layer on sharp bodies was examined in [10-13]. The angles of limiting stream lines computed in [10] and the velocity profiles on a circular cone at angle of attack were compared with experiment in [11]. The friction factor distributions were measured on ogive-cylindrical bodies in [12, 13].

The present paper describes a statement of the problem and computing algorithms for the compressible laminar boundary layer on a sharp body. Computed results are presented for a body of bielliptic cross section at Mach number $M_\infty = 2$ and angles of attack $\alpha = 0\text{-}10°$. The evolution of the three-dimensional boundary layer with variation of angle of attack is described.

1. We consider flow over a sharp body of fuselage shape, immersed in a supersonic stream of gas of Mach number M_∞. The body has a plane of symmetry, which contains the velocity vector of the incident flow. The vector makes the angle of attack α with a certain axis of the body. In this case the entire flow also has a plane of symmetry.

The body surface is assumed to be smooth, and its equation is given in a cylindrical coordinate system $r = r(\xi, \zeta)$. The coordinate ξ is reckoned from the body vertex along its axis, ζ is the meridional angle in the transverse section, and $\zeta = 0$ corresponds to the windward symmetry plane. The equations of the three-dimensional compressible laminar boundary layer have been written in the nonorthogonal coordinate system (ξ, η, ζ), fixed in the body surface [14]. The coordinate η coincides with the local surface normal.

The body nose is assumed to be conical. In that case the inviscid flow there is conical, and the boundary layer equations have a similarity solution dependent on the variables ζ, $\lambda = \eta/\sqrt{\xi}$ [15]. Therefore in this paper, in addition to the coordinate η we introduce the variable λ, and instead of the components of the velocity v directed along the normal to the body surface, we introduce the mass flux

$$j = \rho(v\sqrt{\xi} - u\eta/2\sqrt{g_{11}}).$$

Here and below g_{ik} are the metric coefficients of the surface. Of course, with this substitution we can avoid the solution depending on the longitudinal coordinate ξ only on the conical nose. On the rest of the surface the dependence of the boundary layer thickness on ξ in the new variable λ will be weaker.

We now write two equations of motion, the energy equation, and the continuity equation in the variables (ξ, λ, ζ) in the following form [16]:

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 55-61, January-February, 1991. Original article submitted May 19, 1989; revision submitted July 7, 1989.

